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Observations of oscillatory motion in certain swirling flows

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A descriptive experimental study was made in both air and water of the temporally periodic motion that occurs in the vortex whistle and cyclone separator. The motion can be described in terms of an oscillator that derives its energy from hydrodynamic instability of the steady swirling flow and whose frequency is determined by an angular velocity characteristic of this steady flow. The relevant dynamical parameters are the Rossby number and Reynolds number for the steady flow with the addition of the Strouhal number for the time-dependent flow. The results of this study were compared with the vortex breakdown phenomenon over swept-back wings. Breakdown can be described in the same terms as for the other two cases and it appears that all three motions are basically the same.

1. Introduction

Time-dependent periodic motion has been observed by others in three cases where swirling flow is present, but as yet no detailed explanation has been given. It was the purpose of this experimental study to determine the nature of the motion in two of these cases and to discover whether there is any similarity between the several motions. The flows to be described may be classed as those without a rigid boundary near the swirl axis and therefore need not be related necessarily to those observed between concentric rotating cylinders. The term 'periodic motion' will be used in this paper to imply temporal periodicity; spatial periodicity may occur as a result of this temporal motion.

The first case explored in the experimental study was that of the 'vortex whistle'. The sound in this case is a by-product of the periodic motion and will not be discussed further. A cross-section of the geometry is shown in figure 1. The fluid first passes through a swirl generator, then down the tube and into a medium of similar fluid. Near the exit plane periodic motion of the fluid has been observed. In earlier experiments the swirl was generated by a single tangential inlet to a tube whose diameter D was larger than the diameter d of the downstream tube. In the present study this type of swirl generator was supplemented by three others.

The second case explored was that of the cyclone separator, a device used primarily for dust collection. A cross-section of the geometry is shown in figure 2. Again the fluid passes through a swirl generator which usually has a single tangential inlet and then into the tube. In this case the fluid is restrained from passing down the tube by a back wall and must leave through a smaller concentric discharge tube at the same end as the swirl generator. In commercial separators the back wall usually has a conical shape, as shown in the figure, to aid in the collection of the separated particles. The axis of the separator normally is vertical with the back wall at the bottom.

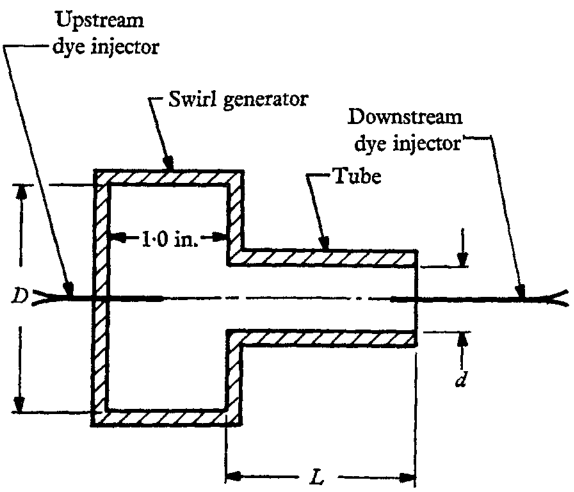


FIGURE 1. A cross-section of the vortex whistle. Four different swirl generators were used; the 1·0 in. dimension applies only to those with tangential inlets.

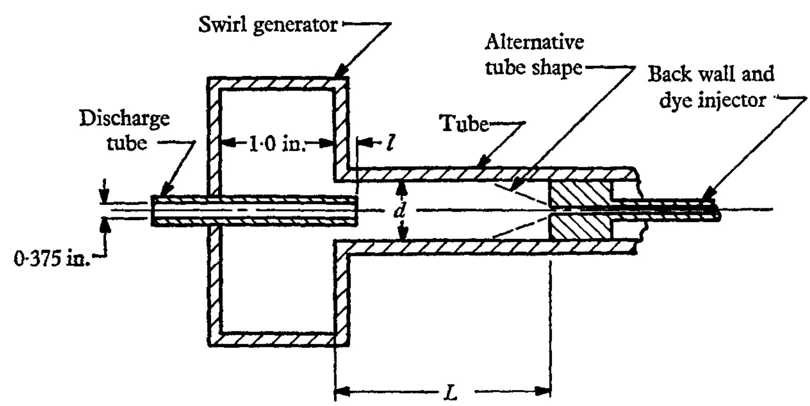


FIGURE 2. A cross-section of the cyclone separator model. Both discharge tube and back wall could be moved axially.

It might be noted that a third geometry can be constructed from either the vortex whistle or the cyclone separator. Adding suction on the axis of the swirl generator in the whistle or removing the back wall and making the tube of finite length in the cyclone separator can form this geometry. It was studied as a special case of the cyclone separator.

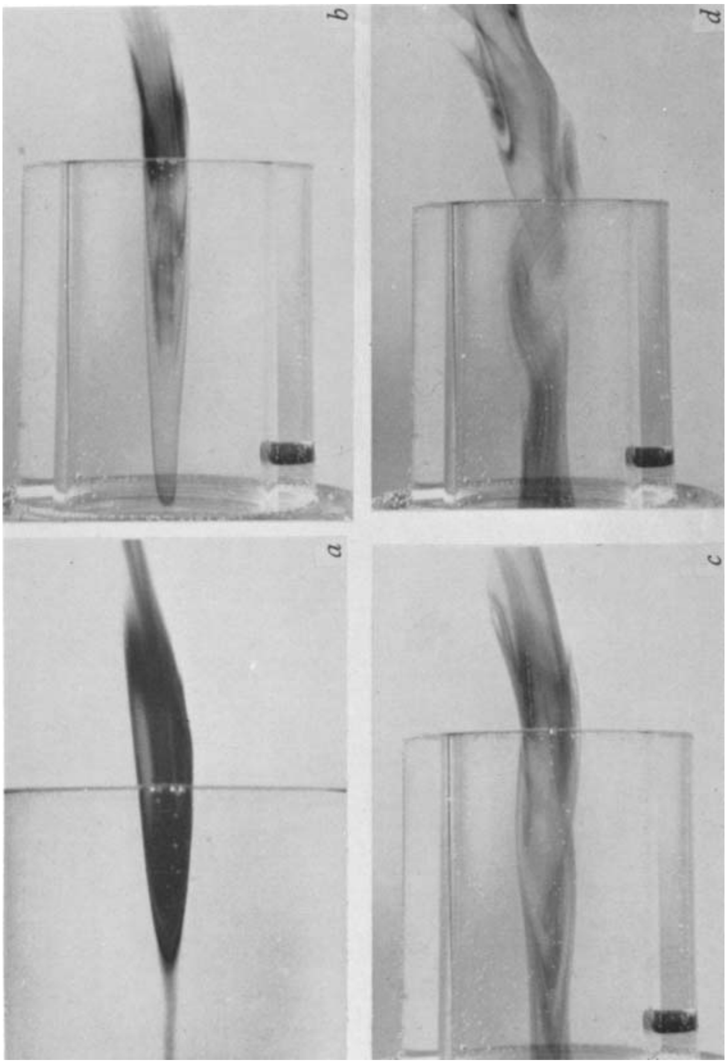
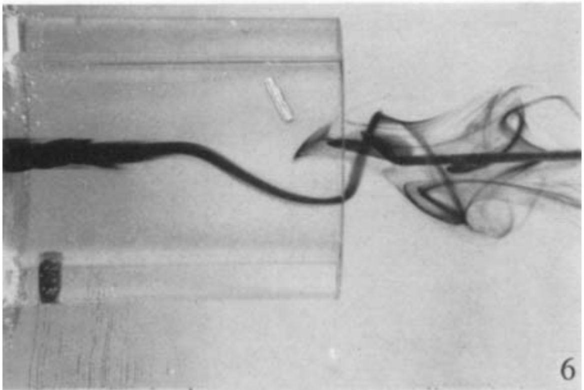
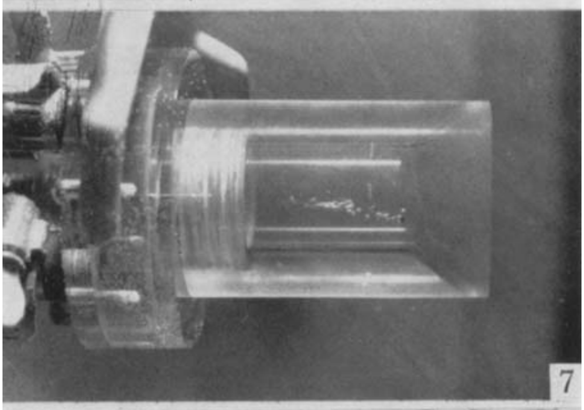


FIGURE 4. (*a*) A photograph of the steady reversed flow region in the vortex whistle with dye in water. (*b*) The reversed flow region at a slightly higher Reynolds number. (*c*) The reversed flow region near R_{cr}. A slight spiral pattern of the dye can be seen. (*d*) The periodic motion when established. A well-defined spiral centre of rotation occurs along with a reversed flow of dye on the axis.





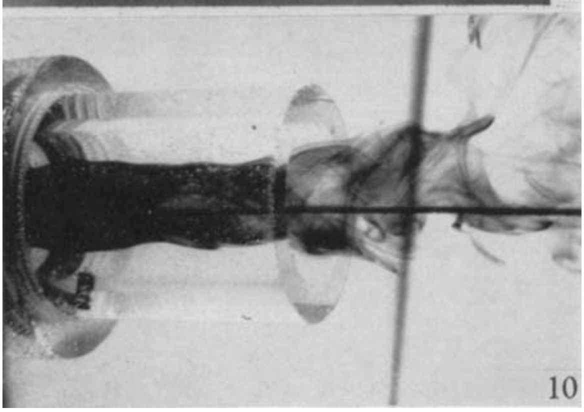
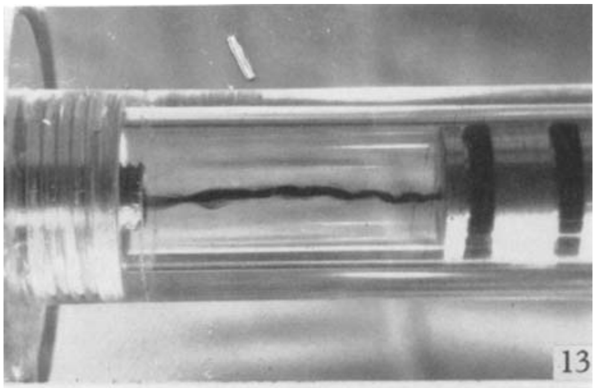


FIGURE 6. A photograph of the large amplitude motion in the vortex whistle showing the reversed flow of dye injected near the axis.

FIGURE 7. A photograph of air bubbles trapped on the precessing swirl axis at higher Reynolds numbers in the vortex whistle.

FIGURE 10. A photograph of the finite amplitude motion as a result of the second non-axisymmetric mode of oscillation. The motion appears to be due to two rotating intertwined spiral vortices.



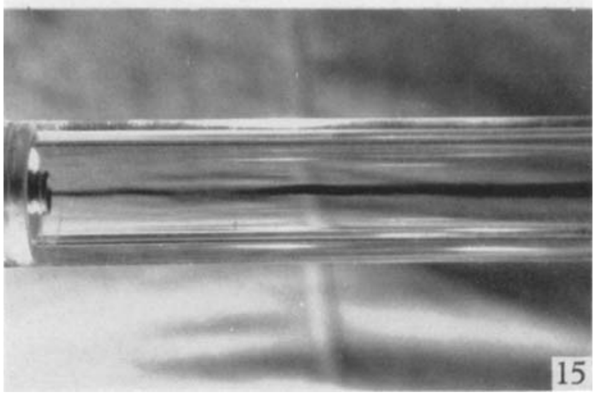


FIGURE 13. A photograph of the steady flow in short separators using dye in water.

FIGURE 14. A photograph of the periodic motion in longer separators using a small air column on the swirl axis.

FIGURE 15. A photograph of the steady flow in longer separators achieved by an increase of flow rate from that of figure 14.

The third case where periodic motion has been observed is that associated with the separated flow on swept-back wings of high-speed aircraft. Because of the high sweep back of the leading edge, a swirling flow is generated above the upper surface of the wing when the angle of attack is sufficient to cause separation. Periodic motion has been observed on the swirl axis at various distances downstream of the leading edge. No study was made of this geometry since there is sufficient experimental evidence available to make a qualitative comparison of this motion with that reported here.

As with all rotating flows, the three important dynamical parameters for low-speed flows are the Reynolds number Wd/ν, the Rossby number W/f_0d and the dimensionless frequency f/f_0 for time-dependent motions (Howard 1963). For the present experiments, W is the mean axial velocity in the tube; d is the tube diameter; ν is the kinematic viscosity; f_0 is a frequency characteristic of the angular velocity and f is the frequency of the periodic motion. In addition, the dimensionless tube length L/d will be used.

2. Previous work

The previous theoretical and experimental work pertinent to each case will be discussed separately; the relationship between the vortex whistle and cyclone separator will become apparent in § 3 and the relevance to vortex breakdown will be discussed in § 4.

2.1. *The vortex whistle*

Vonnegut (1954) was apparently the first to report on this phenomenon in the literature. His whistle was similar to that shown in figure 1; it had a one-tangential inlet swirl-generator whose diameter, D, was greater than that of the tube. With air as the fluid, he detected a tone whose frequency increased with increasing flow rate. With water as the fluid, he observed that small air bubbles, trapped on the swirl axis in the tube, precessed at a well-defined frequency if they were near the exit plane. He suggested that the precession velocity was about the same as the fluid angular velocity. He made no attempt to explain the motion. Michelson (1955) later examined the inviscid equations of motion of two-dimensional flow for this geometry and deduced that there was an allowable frequency which depended on a characteristic flow velocity.

Later experiments (Chanaud 1963), primarily concerned with the acoustical aspects, have shown that the precessional frequency is identical to the acoustical one and have strongly reinforced the supposition that the precessional angular velocity is related to the fluid angular velocity. The measurable parameters found useful in describing the motion were the Reynolds number, as previously defined, and a Strouhal number fd/W which is the ratio of the dimensionless frequency to the Rossby number. This Strouhal number was found to be of the order of unity. The supposition that the dimensionless frequency also be of order one requires that the Rossby number be of similar order when this motion occurs. Gore & Ranz (1964) observed oscillations in a swirling flow similar to that in the vortex whistle but did not describe their nature. Their Rossby number was of the order of one.

Suzuki (1960), although primarily concerned with temperature-separation effects in swirling flow, also made observations on the vortex whistle. He apparently thought the periodic motion was a result of hydrodynamic instability since he did a small perturbation analysis of the inviscid equations of motion. The velocity disturbances were of the form $u' = u(r)\exp[i(\sigma t + m\theta + kz)]$ where σ was the complex frequency, k the wave-number in the axial direction and m the wave-number (an integer) in the angular direction. The steady flow model in the analysis was divided into two regions: a solid body rotational core with uniform axial velocity and an outer region with a free vortex distribution and zero axial velocity. His analysis suggested he knew that in the real case k was negative and m was equal to minus unity. He deduced that when $k = 0$ the perturbation frequency was equal to the angular velocity of the fluid in the solid body core or at least linearly related if $m \neq -1$. From his experimental observations of sound frequency, he implied that this theoretical deduction is valid although he apparently did not measure the angular velocity of the steady flow. He noted that insertion of a rod along the tube axis could stop the tone emission.

Relevant to the concept of hydrodynamic instability as the cause of the periodic motion is the work of Howard & Gupta (1962). They considered the general problem of the stability of inviscid two-dimensional swirling flows to non-axisymmetric disturbances of the same form as used by Suzuki. It was deduced from their stability equation that Rayleigh's criterion applies for the case $k = 0$; the presence of an axial velocity is then irrelevant. The criterion states that for stability the vorticity of the tangential component of the swirl must increase monotonically with radius. They were unable to arrive at any general criteria for the case $k \neq 0$ with arbitrary velocity profiles. They gave as a sufficient condition for stability:

$$(2k\Omega/r)\,[kd(r^2\Omega)/dr - mdW/dr] - \tfrac{1}{4}[kdW/dr + md\Omega/dr]^2 \geqslant 0 \text{ everywhere.} \quad (1)$$

Ω is the angular velocity of the steady motion. Both important components of vorticity are now included. For a real swirling flow without an inner boundary, $\Omega \simeq$ constant near the axis so the stability condition would reduce to $(2k\Omega)^2$ there, i.e., stability, only if dW/dr approached zero faster than r. For the real velocity distribution that occurs in the tube of the vortex whistle, this stability condition achieves a high negative value near the tube wall implying instability, but it is also here that viscous effects are most important and so one could not expect the inviscid equations to be relevant. The frequency term in their stability equation was of the form $\sigma + kW + m\Omega$. An unstable solution of the type suggested by Suzuki, i.e. $\sigma_r = \Omega$, $m = -1$, could only occur, for the case $k \neq 0$, if $W = 0$. These conditions will be re-examined in § 4.

2.2. *The cyclone separator*

Smith (1962*a*, *b*) studied the details of the flow in a separator model similar to that shown in figure 2, i.e. there was no taper in the main tube. He observed and photographed a periodic flow regime. He described it by stating 'that the vortex, instead of being attached to the center of the bottom plate, turned and attached itself to the cylindrical wall of the cyclone. The point of attachment rotated in a horizontal plane'. He noted that once this periodic motion was established it was quite stable and disappeared only at 'negligible' flow velocities. The frequency of this motion was observed to be dependent on flow rate through the separator, although the type of dependence was not stated. With increasing flow rate this point of attachment was observed to move towards the back wall. When within two diameters of the back wall, the point of attachment jumped abruptly to the centre of the wall and the flow became steady. He noted that this periodic flow was never observed in a cyclone shorter than four tube diameters and was always observed in those longer than 12 diameters. Sudden addition of dust was observed to cause development of this periodic motion. N. W. Eft, a discusser of Smith's papers, noted that a conical tube appeared to help maintain a steady flow that was stable.

2.3. *Vortex breakdown*

The term 'vortex bursting' or 'vortex breakdown' has been applied to describe the structural change that occurs in the swirling flow generated on the upper surface of a lifting delta wing or one with high sweep back when the angle of attack of the wing is sufficient to cause separation at the leading edge. Two opposing swirl flows are generated on either side of the axis of symmetry of the wing but appear not to influence each other. This change appears spatially in two stages. The steady swirling flow first alters to a periodic spiral motion at some point along the swirl axis and then further downstream it changes to an irregular turbulent motion. Lambourne & Bryer (1962) have taken photographs in laminar flow that clearly show these changes.

Theory and experiment associated with this phenomenon have led to two somewhat different explanations. The first is that breakdown is due to the amplification of small disturbances and the second is that it is an abrupt change between two basic types of rotating flows.

The first explanation is supported by Squire (1960) who suggested on theoretical grounds that breakdown may be caused by spatial amplification of small axisymmetric disturbances (standing waves). Time dependence was not considered. He further suggested that a critical value of the ratio of rotational velocity to axial velocity may exist. If the rotational velocity can be characterized by an angular velocity and length scale, then this ratio is the reciprocal Rossby number. The experiments of Lambourne & Bryer have shown that there was, in fact, a spatial growth of disturbances within the swirling flow but that it was spiral, i.e. non-axisymmetric, and temporally periodic as well as spatially periodic. Within the spiral flow an axial flow reversal was observed. They were able to measure the frequency and found it increased with increasing flow rate. This is strong evidence in favour of the amplification explanation. They noted, however, that the position of the breakdown was not sensitive to small disturbances to the vortex and surmised from this that the phenomenon may not be dependent on amplification of small disturbances.

Harvey (1962) performed experiments on the swirling flow within a long cylindrical tube and observed a region of reversed flow on the axis; it had the form of a sphere at one condition. The position of this region was maintained by careful adjustment of the inlet angle of the peripheral vanes in the swirl generator. His photographs show no spiral structure similar to that observed over wings. He considered this flow reversal as indicative of a critical phenomenon. Benjamin (1962) gave theoretical support to this conclusion by considering the change of flow direction on the axis as analogous to the hydraulic-jump phenomenon and proposed his theory to be relevant to the vortex breakdown over delta wings. This latter explanation may be revelant to Harvey's case but for breakdown it is not complete, although Benjamin stated that the spiralling over wings was probably attributable to 'lateral fluctuations superposed on a steady axisymmetric configuration'. Gartshore (1962) suggested a somewhat intermediate explanation. He considered the vortex breakdown, characterized by flow reversal, as due to vorticity diffusion but added that the flow downstream of this reversal may be unstable. He proposed that the observed spiral flow is evidence of this instability.

3. The experimental observations

Observations were made in both air and water. In the air system, the fluid passed from the building supply through several pressure regulators and then through one of several carefully calibrated orifice meters. It then passed through a small silencer to eliminate the separation noise of the orifice plate and through the inlet hose to the model of interest. In the water system, the fluid was obtained from a constant head tank and passed through one of two carefully calibrated rotameters. It then passed through the inlet hose to the model and thence into a relatively large water tank which had two Lucite side walls. Fluid temperatures were measured just beyond the model exits.

The models shown in figures 1 and 2 were made of Lucite and the tube diameter d in each case was 1 in. Visual observation of the flow in water was facilitated by injection of both dye and small flat aluminium particles. The size of these particles required a ten power binocular microscope to trace their motion. The normal positions of the injectors are shown in the figures; they were hypodermic tubes of 0·05 in. outside diameter and had no observable effect on the flow.

Measurement of frequency in the air system was made with either a Disa constant-temperature hot-wire anemometer or a Bruel and Kjaer microphone, type 4133, in conjunction with a Computer Measurements Co. frequency counter. In the water system this measurement was made at low flow rates with a stopwatch and hand counter by observing dye motions and at higher flow rates with a stroboscope and frequency counter by observing the motion of small air bubbles on the swirl axis.

There were four types of swirl generators used in the experiments. The first had a single tangential inlet of circular cross-section, 0·5 in. diameter, to a tube whose diameter D was 2·5 in. The second had four tangential inlets of rectangular cross-section, 1·0 in. by 0·25 in. to a tube whose diameter D was 1·0 in. The third had a set of eight stationary swirl blades set directly into the 1·0 in. tube, whose exit angles were approximately 60° to the tube axis. The blades had no twist and extended almost to the tube axis; the centre body was a dye injector. The fourth was a rotating section of 1·0 in. tube upstream of the fixed tube. In this last swirl generator, the fluid entered a straight section of fixed tube, 20 in. long, to develop a nearly parabolic velocity profile. It then passed through a section of rotating tube, 30 in. long, by which swirl was generated, and then passed through another section of fixed tube into the ambient medium. It was the length of this latter fixed tube that was used as the characteristic length L. A Zero-Max variable-speed drive was used to rotate the moveable tube. The frequency of rotation was deduced from counting the revolutions, as shown by an attached counter, over a known time period.

Lay (1959) suggests that tangential inlets produce tangential velocity profiles in the swirl generator that are more nearly like those of a potential vortex and that these profiles tend to change in the downstream direction to ones with a finite sized solid-body rotational core. The swirl generator with a rotating wall caused a tangential velocity profile more like that of total solid-body rotation. The swirl blades probably generated a tangential velocity profile between these extremes.

3.1. *The vortex whistle*

Initially it was considered important to observe, qualitatively, the details of the flow near the tube exit as a function of flow rate. To accomplish this, the swirl generator with four tangential inlets was attached to a tube of length $L/d = 4{\cdot}0$ and the system was operated in water with both dye and particles together rendering portions of the fluid visible. The following paragraphs describe what was observed as the flow rate was slowly increased from very low Reynolds numbers $R = Wd/\nu$.

For $R < 300$, the swirling flow was laminar and steady. The motion beyond the exit decayed in a regular manner due to diffusion of vorticity. Observation of particle motion in the tube clearly showed that the angular velocity was greatest near the axis and there appeared to be a core of diameter $d/4$ in solid-body rotation. The axial velocity was much greater near the tube wall than near the axis. At $R \simeq 300$ a small region of reversed steady flow was observed to form just downstream of the exit of the whistle. It was centred on the tube axis and was more elongated than the reversed-flow region photographed by Harvey (1962). With a slight increase in flow rate, the free stagnation point moved to a position upstream of the exit and the region lengthened somewhat in extent. The axial flow pattern associated with this condition is shown in figure 3. Dye in this region showed what appeared to be a ring vortex rotating in its own plane. Because of dye diffusion it was not clear whether this region was completely self-enclosed. Further increases of flow rate caused the stagnation point to move further upstream until at $R \simeq 325$ the stagnation point was about two diameters upstream of the exit. In this condition, the flow was disturbed impulsively by rapidly squeezing one of the four inlet hoses. Dye was being introduced by the upstream injector and disturbances to this dye were observed to move down the tube and, when they encountered the reversed flow region, a regular temporally periodic motion, as described below, was seen. After several cycles the motion would evanesce. With only an exceedingly small increase in flow rate, this periodic dye pattern took longer to decay when impulsively caused. With a further slight increase of flow rate, this pattern remained once caused. Generally, another slight increase of flow rate was necessary to cause the periodic motion to start naturally from ambient disturbances. The 'slight increases' referred to here were too small to be measured and were inferred from small changes in the steady dye pattern. When once established, the motion soon became of finite amplitude and the dye proceeding downstream on the axis left it near the stagnation point and formed a rotating spiral pattern which became irregular about one diameter downstream of the exit. Repeated duplication of this event with additional dye streams at various radii showed there was virtually no disturbance slightly beyond the stream surface which enclosed the steady reversed-flow region (see figure 3). A subjective impression obtained of the rotating-spiral-dye pattern was that the dye disturbances grew close downstream of the stagnation point and the resultant pattern was simply convected over the bounding stream surface. For a discussion of the difficulties in interpretation of dye patterns, see Hama (1962).

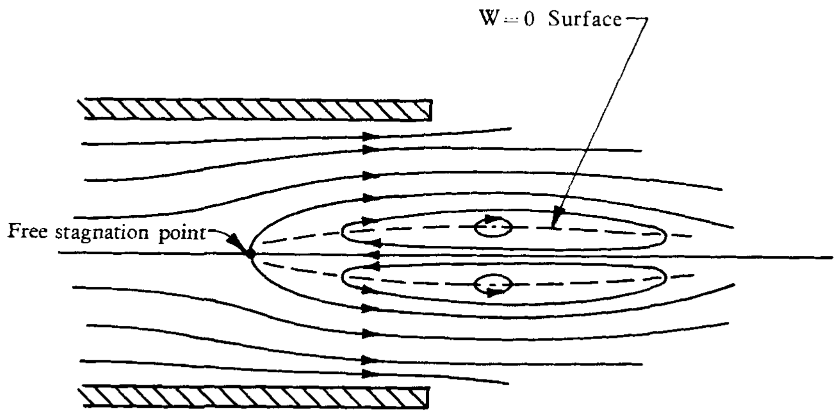


FIGURE 3. The steady axial flow pattern at the vortex whistle exit at low Reynolds numbers. Periodic motion would not commence without this reversed flow.

It is evident that this phenomenon possesses a 'critical' Reynolds number, denoted as R_{cr} above which small disturbances are amplified. Further, since the periodic motion was exceedingly stable and regular, observation left no doubt that this flow was a most singular hydrodynamic oscillator: one completely embedded in the fluid itself and only indirectly tied to some bounding rigid surface.

Because the motion was sufficiently slow, it was possible to observe that the first few cycles, i.e. when the motion was barely detectable, had a period only slightly greater than the rotational period of the steady particle motion in the core. After several cycles, the motion had increased in amplitude and had significantly influenced the downstream flow field and it was then observed that the period of the motion had increased to about 1·4 times its initial value. Further increase of flow rate over that necessary to just cause the motion made the stagnation point move further upstream until it encountered the back wall of the swirl-generator. The axial position where disturbances first became visible moved upstream only very little. The motion then need not bear a fixed relation to the free stagnation point.

It became clear that the oscillation, when of somewhat greater amplitude than that described above, was the first mode of a non-axisymmetric disturbance whose phase surface moved angularly in the direction of fluid rotation and axially in the direction of the mean flow. In the notation used earlier, $m = -1$ and k must be replaced by $-k$. This confirms the correctness of Suzuki's use of these values. Near R_{cr}, $k \simeq 2\pi/d$.

To illustrate the above described sequence, a series of photographs was taken and is shown in figure 4, plate 1. Each represents increasingly higher flow rates. In figure 4(*a*) the reversed-flow region had just formed near the tube exit. The flow was from left to right; the dye was introduced on the axis by the upstream injector. In figure 4(*b*) the reversed flow region had elongated, predominantly in the upstream direction. In figure 4(*c*) the flow rate was slightly above the critical and a spiral displacement of the dye barely can be seen. In figure 4(*d*) the motion was of somewhat greater amplitude; the free stagnation point had moved to the back wall of the swirl generator.

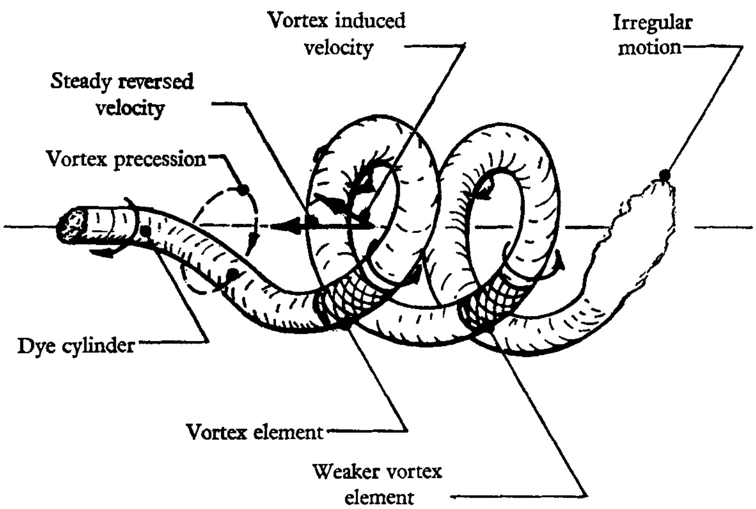


FIGURE 5. A sketch of the periodic motion in the vortex whistle at large amplitude as deduced from observation. The motion is that of a rotating spiral vortex.

At higher Reynolds numbers, the periodic motion was of large amplitude and, when dye was injected near the axis from upstream, a flow such as drawn in figure 5 was observed. The shaded region represents a cylindrical sheet of dye which had moved from upstream and then deformed into a rotating spiral pattern. It was quite evident after repeated observations that at this amplitude the dye represented the central region of a precessing spiral vortex. Figure 4(*d*) is also suggestive of this motion. The weaker vortex element downstream was forced to move in the velocity field of the upstream element. This was an apparently unstable condition as disruption to irregular motion occurred in a short distance. This spiral vortex induced a precessing velocity near the axis which was superposed on the reversed axial flow. A photograph of the motion at this Reynolds number is shown in figure 6, plate 2. The downstream dye injector was present in this photograph and the dye from it was moving upstream in an oscillatory manner. At still higher flow rates small air bubbles could be trapped on the precessing swirl axis. Figure 7, plate 2, shows this condition with a conically increasing area at the exit. It should be noted here that figures 5 and 6 of this paper are remarkably similar to figures 5 (*b*) and 8, respectively, in the report on vortex breakdown by Lambourne & Bryer (1962).

Certain changes in geometry of the whistle were made. First, the tube length was varied and it was found that the critical Reynolds number was higher for longer tubes. For example, with $L/d = 2{\cdot}0$, $R_{cr} = 270$ and, with $L/d = 9{\cdot}0$, R_{cr} was well into the thousands. Since the tube primarily causes decay of the tangential component of velocity, the Rossby number increases with increasing axial position. For a given Reynolds number then there is a Rossby number above which periodic motion will not occur. When the tube exit had a conical contour (as shown in figure 7) whose cross-sectional area increased towards the exit plane, R_{cr} was found to be less than for a straight tube of the same length. When the tube exit had a conically decreasing cross-sectional area to an exit of 0·5 in. diameter, R_{cr} increased to a value beyond the capacity of the rotameters. A large flat plate, held perpendicular to the axis of the whistle with a straight-sided tube, caused a reduction of R_{cr} when within one diameter downstream of the exit plane. Presumably, the momentum defect in the boundary layer on this plate was sufficient to cause a radial inflow there which in turn caused the reversed flow to occur at a lower Reynolds number (Bodewadt 1940). The presence of a reversed-flow region then is important.

With the straight-sided tube, 4 diameters long, the reversed-flow region was established at a Reynolds number slightly below the critical one. The downstream dye injector was turned on sufficiently to make the injected dye have greater momentum than the fluid in the reversed region into which it flowed. The periodic motion commenced. The upstream injector was then used to extract approximately the same amount of fluid as injected downstream and the periodic motion ceased. This suggests that the velocity profiles near the axis play an important role in determination of R_{cr}.

A rod was inserted on the tube axis from the back wall of the swirl generator to 3 diameters beyond the whistle exit. When the rod diameter was 0·20 in., there was no observed effect on R_{cr} or on the form or period of motion, when established. A rod diameter of 0·375 in. was found to eliminate the motion entirely.

To test dynamic similarity based on the Strouhal number, as defined in § 2.1, and the Reynolds number, three of the swirl generators were used in both air and water to obtain frequency data. The results are shown in figure 8. The swirl flows generated by the tangential inlets do not display close similarity. The difference is greatest for the generator with the larger ratio of generator diameter to tube diameter. This result was completely unexpected and no adequate explanation can be suggested at present.

As a result of the previous experiments, it was anticipated that there would be a range of Rossby numbers in which the motion could develop and that there would be a minimum critical Reynolds number below which no periodic motion would occur at any Rossby number. It was for verification of this that the swirl generator with a rotating tube was made. Water was used and dye was introduced on the tube axis through a long hypodermic which terminated about one diameter downstream of the rotor. Data were taken by holding the flow rate constant and increasing the rate of tube revolutions. The reciprocal of the Rossby number was used to make the resultant stability diagram similar to the usual ones, i.e. increasing frequency with increasing ordinate. Gore & Ranz (1964) have called this reciprocal the 'swirl ratio'. The results are presented in figure 9 for a tube with $L/d = 3{\cdot}0$ and are explained below.

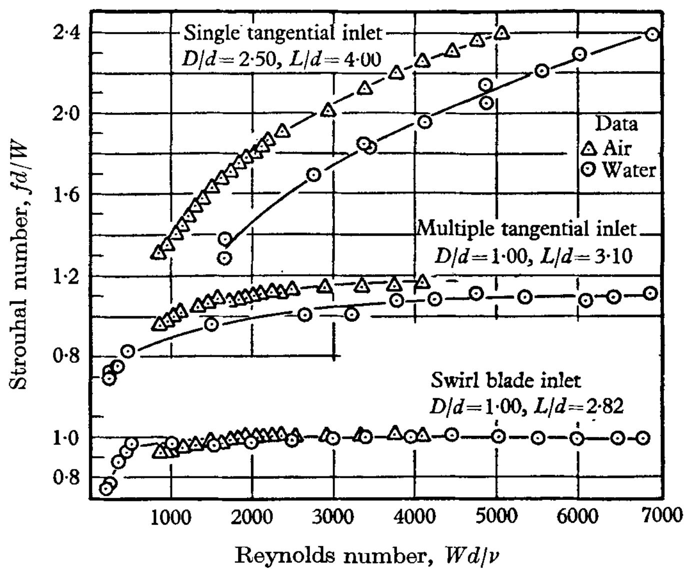


FIGURE 8. Dynamic similarity for the vortex whistle holds when the swirl generator causes a constant Rossby number flow at any Reynolds number.

At $R < 200$, a reversed-flow region was found to occur about 1 diameter downstream of the end of the rotating wall at a certain rate of revolutions and was slightly more elongated than that observed by Harvey (1962) under different conditions. With increasing rate of revolution, the region spread both upstream and downstream until there was a reversed flow core beyond the tube exit and well into the rotor. At $R \simeq 200$, two separate reversed flow regions occurred, one at the rotor and one at the tube-exit. With increasing rate of revolution, these regions merged to form an elongated reversed-flow region as at lower Reynolds numbers. At $R \simeq 245$, a faint periodic motion was observed; this can be considered the minimum critical Reynolds number for a tube of length $L/d = 3{\cdot}0$. At higher Reynolds numbers the region of periodic motion encompassed an increased range of Rossby numbers. The lower limit (lowest frequency) was well defined but the upper frequency limit was quite difficult to obtain, hence the dashed line in the figure. Near this upper limit there appeared to be an irregular disturbance, slightly downstream of the rotor, which interfered with the periodic motion near the exit. It was later observed that this disturbance was the second non-axisymmetric mode of oscillation. The first and second mode would alternately appear in an apparently random manner. The finite amplitude motion of this second mode resulted in two inter-twined rotating spiral vortices somewhat upstream of the tube exit! A photograph of this remarkable pattern is shown in figure 10, plate 2; a better photograph could not be obtained.

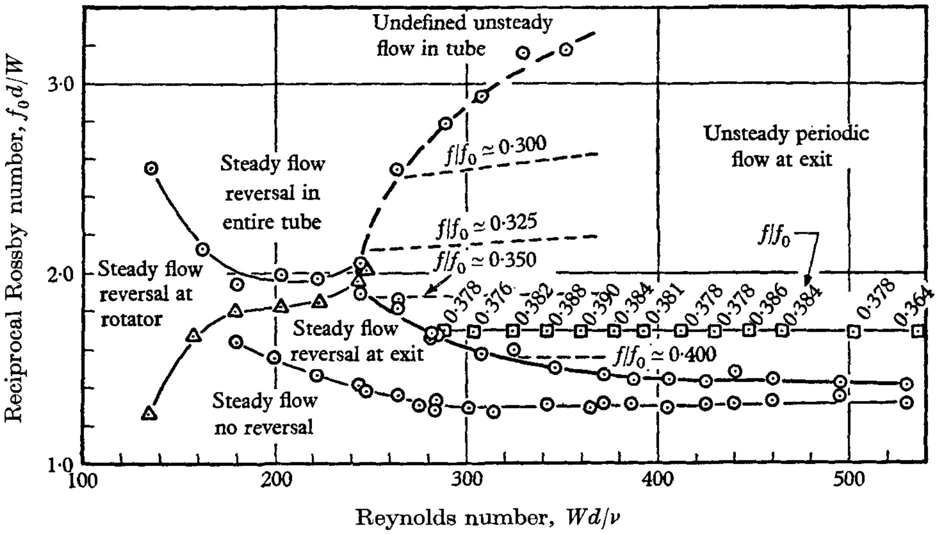


FIGURE 9. The limits of periodic motion in the vortex whistle can be described in terms of the Rossby number and Reynolds number and there exists a minimum critical Reynolds number. Flow reversal occurs only near the axis.

Approximate contours of the dimensionless frequency are shown in figure 9 and for one particular Rossby number the dimensionless frequency was found as a function of Reynolds number; it can be seen that the frequency of the periodic motion bears a constant relationship to the angular velocity of the steady flow. The result confirms the original supposition made by Vonnegut (1954). The data were not extended to higher Reynolds numbers since one could no longer be sure that the angular velocity of the fluid bore a fixed relationship to the angular velocity of the rotating tube. It should be remarked that the dimensionless frequency deduced here is not the same as the one noted earlier, i.e. $f/f_0 = 1/1{\cdot}4 \simeq 0{\cdot}7$. The fluid in the rotating-tube experiment apparently did not acquire the angular velocity of the tube wall but it was this latter quantity that was used in the calculations. The decrease of the calculated dimensionless frequency with increase of the rate of revolution of the wall tends to corroborate this expectation.

3.2. *The cyclone separator*

In these experiments, the model of figure 2 was used in water with the swirl generator having four tangential inlets. Dye could be injected through the back wall and through one of the inlet hoses to render the flow near the discharge tube visible.

The steady axial flow pattern within the tube was deduced from observations of both dye and particles in laminar flow and is drawn in figure 11. It is quite similar to that deduced by Smith (1962*a*, *b*) from velocity measurements at much higher flow rates. It can be seen that the flow enters the tube in an annulus and is forced toward the axis as it reverses direction to leave finally through the concentric discharge tube. At slightly higher flow rates small dye oscillations were observed near the $W = 0$ surface. Because it was difficult to interpret visually dye patterns in this model, it was not possible to draw a descriptive diagram similar to figure 5. It was clear, however, that the motion was a non-axisymmetric oscillation and that the displacement of the dye increased towards the back wall.

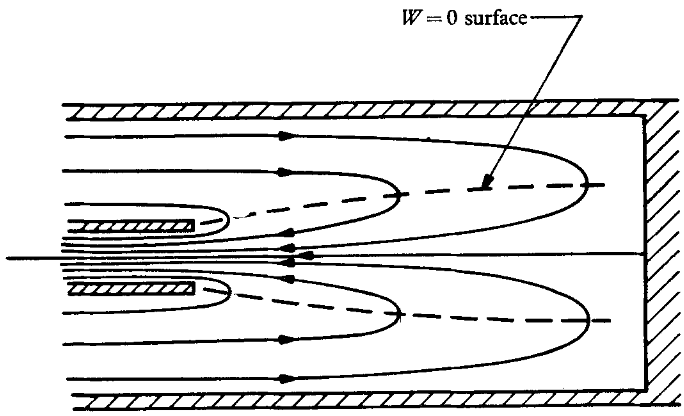


FIGURE 11. The steady axial flow pattern within the cyclone separator model. Unlike the case of the vortex whistle, there is no free stagnation point.

Data were taken to determine the limits of this periodic motion as a function of tube length L and the results are presented in figure 12 for tube lengths in the range used for commercial applications. The Reynolds number was used for convenient comparison with the data of the vortex whistle but should not be interpreted as having necessarily the correct characteristic length scale. The critical flow rate was very close to that for the vortex whistle and was independent of tube length except for very short tubes. It was dependent on the position l of the discharge tube as shown in the figure. The high flow-rate limit for the motion depended strongly on tube length, the region of periodic motion being larger for longer tubes, but did not depend on the position of the discharge tube. At one tube length, $L/d = 4{\cdot}75$, the geometry of the tube was changed to one which conically tapered to 0·5 in. diameter at the back wall. The angle of taper was varied so that the cone started at 1, 2 and 3 diameters from the back wall. The longer cones had increasing influence on the upper limit of the motion; the smallest region of periodic motion occurred with the longest cone. The critical flow rate did not depend on cone length.

To illustrate the meaning of the high flow rate limit, photographs of the motion were taken. Figure 13, plate 3, shows the steady flow in short tubes; the flow condition with respect to the stability limits is shown in figure 12. For the longer tubes at higher flow rates, it was possible to entrain air momentarily on the swirl axis in the periodic-flow region. A photograph of this air column is shown in figure 14, plate 3, and the flow condition with respect to the upper limit is shown in figure 12. Its motion was of the same type as the air bubbles shown in figure 7 for the vortex whistle and was also the same as that described by Smith (1962*a*) in his model of a cyclone separator. The frequency of the motion was observed to be higher at higher flow rates. In addition to Smith's observations, it was noted that, as the point of maximum displacement moved towards the back wall

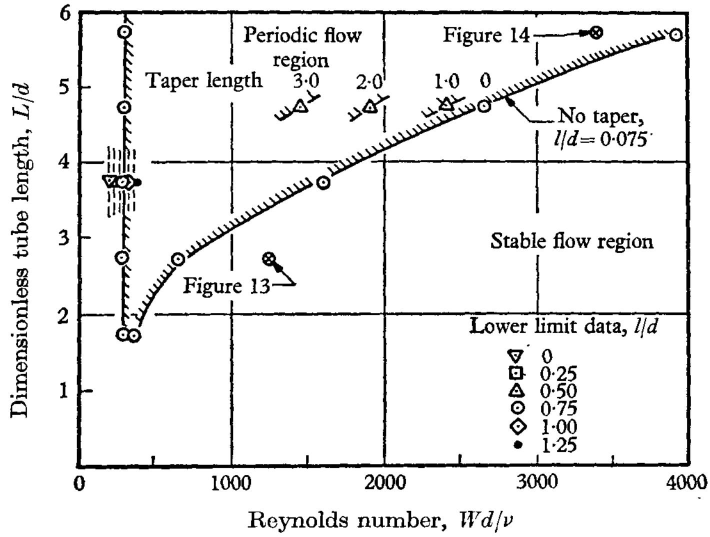


FIGURE 12. The limits of periodic motion in the cyclone separator are functions of flow rate and separator geometry.

with increasing flow rate, the point of minimum displacement did also. The minimum displacement point can be defined as the point where deviation of the swirl axis from the geometric axis just can be detected. This suggests that the periodic motion in the separator need not bear a fixed relationship to any geometric element. When near the back wall, the swirl axis abruptly jumped to the geometric axis and the periodic motion disappeared. A photograph of this steady condition is shown in figure 15, plate 3.

At a fixed flow rate, movement of the back wall produced a hysteresis effect. If the tube length was slowly reduced from a great value, the periodic motion would abruptly disappear at a certain length. If the tube length was then slowly increased, the periodic motion would not reappear until the length was approximately one diameter greater.

The back wall of the separator was removed and the tube length was reduced to 6 diameters to form the intermediate geometry between vortex whistle and cyclone separator as described in the Introduction. When no suction was applied to the discharge tube (upstream), the usual motion of the vortex whistle occurred at the tube exit. When the suction was sufficiently great to extract more fluid than entered through the tangential inlets, reversed flow occurred throughout the tube and no periodic motion was observed. When the inlet flow was divided between discharge tube and whistle exit, increase of suction would reduce the frequency of the motion at the exit and cause commencement of periodic motion at the entrance to discharge tube. Further increase of suction caused the motion at the exit to cease entirely, presumably because the flow rate there was insufficient to sustain it. When the tube length was reduced to four diameters, at high flow rates increasing suction was observed to cause the periodic motion at the tube exit to move smoothly up the tube towards the entrance of the discharge tube. This was clear evidence that the periodic motion in both the vortex whistle and cyclone separator are essentially the same.

4. Discussion†

That the motion can be called properly an oscillator may be deduced from two observations. A finite disturbance was required, in one instance near R_{cr}, to start the motion and once started the motion would remain even though the disturbance was removed. Well defined motion occurred whether the upstream flow was laminar or turbulent. Both observations suggest feedback control of the motion and thereby an oscillator must be inferred.

Theory and experiment provide some insight into the conditions necessary for amplification of small disturbances. The evidence suggests that one focus attention on the possibility that solutions to the equations formulated by Howard & Gupta (1962) may exist and be relevant, especially unstable ones near the condition $\sigma_r = \Omega(R)$, where R is determined by $W(R) = 0$. The two vorticity components must also combine to violate the stability condition of (1). One can deduce from this supposed solution: that a reversed flow is necessary in order to generate both a $W = 0$ surface and a trend toward instability (a positive dW/dr); that a certain value of Rossby number is necessary for instability; that the frequency of oscillation will depend on the angular velocity of the fluid near the $W = 0$ surface; that the motion does not commence on the axis or far from it but near the $W = 0$ surface; that a free stagnation point may not be necessary. This supposed solution can explain nicely many of the observations. For example, the case of starting and stopping the motion at a constant flow rate with the two dye injectors can be explained by reasoning that the upstream flowing dye in the reversed-flow core caused a local increase in dW/dr and therefore instability while suction on the upstream injector caused a local radial flow there which was influential in causing an increased angular velocity downstream and therefore a trend to stability. The lack of influence on R_{cr} of a small coaxial rod can be explained by noting that the supposed motion commences at a finite radius.

To apply the theory at all requires that it be applied *locally* and this is equivalent to stating that three-dimensional effects act only to cause the reversed-flow region but have no important bearing on stability. Commencement of oscillations near the swirl axis away from the fixed outer wall suggests that viscous

† Some of this discussion exceeds the limits set by the experimental evidence presented above, especially with regard to vortex breakdown. The motivation for this extension was to call attention to the possibility that a further study of the vortex whistle might provide a means of resolving the present controversy over the mechanism of vortex breakdown.

effects might be left out of the theory except near the supposed critical layer. At the present time no theoretical solutions are available either for the steady or unsteady flow and so the above deductions cannot be put on a firm basis. It is hoped that the close ties between theory and experiment proposed here will provide further encouragement for theoretical analysis especially with regard to the role of the axial wave-number k since the experiments suggest that the manner in which the steady swirl changes axially has importance.

As noted initially, the strength and regularity of the motion in both laminar and turbulent flow suggest that feedback is the controlling influence. Since the frequency is closely related to the fluid angular velocity even for large amplitude motion, this feedback acts primarily to control amplitude. The obvious feedback path is the reversed flow on the axis where, in the case of the vortex whistle, the spiral vortex induces an additional velocity component of the proper character to redisturb the upstream flow. This is indicated in figure 5. In the cyclone separator it was not possible to observe any spiral structure so the additional axial component may not be important.

Two observations made of this periodic motion make it appear possible that it can occur over swept back wings. The first is that the motion need not be directly tied to any geometric element but is dependent rather on the position of the swirl axis and the conditions for flow reversal. This would answer any questions about asymmetry of boundary conditions in the vortex breakdown case. The second is that the motion is of the nature of an oscillator and depends strongly on the feedback of disturbances from downstream. This would answer the question about sensitivity of this motion to outside disturbances.

Observations of vortex breakdown have shown that flow reversal is intimately related to it and that the motion is like a rotating spiral vortex whose frequency increases with increasing flow rate; all these observations are identical to those made of the vortex whistle. The position of breakdown was found to depend primarily on a parameter which can be shown to be like a reciprocal Rossby number and was found to be virtually independent of Reynolds number (Lambourne & Bryer 1962); the importance of these parameters has been established in the case of the vortex whistle. The photographic and diagrammatic similarities between the two phenomena, noted in § 3, provide further strong evidence that the case of vortex breakdown is simply another geometry where this unusual hydrodynamic oscillator can appear.

5. Conclusion

These experiments have shown that the periodic motion in both vortex whistle and cyclone separator can be described in terms of a hydrodynamic oscillator which normally operates in the first non-axisymmetric mode and whose frequency is related closely to the angular velocity of the flow. The important parameters have been shown to be the Reynolds number, Rossby number and dimensionless frequency; the latter two can be replaced conveniently by the Strouhal number. The parameters are all of such magnitude that it appears no important simplifications can be made in the equations of motion to solve the problem analytically. The energy of the oscillator is derived from the hydrodynamic instability of the fluid within a reversed-flow region on the swirl axis. No quantitative information is available on the conditions for formation of a steady reversed-flow region but the experiments suggest that two-dimensional perturbation analysis may have some value in describing the amplifier part of the oscillator. The experiments have shown there is a minimum critical Reynolds number below which no periodic motion will occur at any Rossby number and that, at a given Reynolds number above the critical, there is a Rossby number above which no periodic motion will occur.

Since the periodic motion appears to be controlled by feedback through oscillations superimposed on a reversed mean flow, it is more reminiscent of vortex shedding behind circular cylinders than of systems, such as the edge tone (Powell 1961), that are controlled by purely oscillatory feedback.

The strong similarities of the motion described here to that of vortex breakdown leads the author to favour Gartshore's (1962) estimate as being substantially correct.

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